M.A.-101(Old)

B. Tech. (Semester I) Examination – 2011

Maths - I

Time: Three Hours

Maximum Marks: 100

Note: Attempt question from all the sections.

Section –A

Note : Attempt any ten question. Each question carries four marks. **(4x10=40)**

1. Show that the matrix

$$A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$ is orthogonal

2. Find the ranks of given matrix

$$\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

3. Find out for what values of λ the equation:

$x + y + z = 1$

$x + 2y + 4z = \lambda$

$x + 4y + 10z = \lambda^2$

have a complete solution and solve it

4. If $\mu = e^{xyz}$ Show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

5. Trace the curve

$$ay^2 = x^2(x - a)$$

6. If $\mu = f(r), where\ r^2 = x^2 + y^2$

Prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f^1(r)$$

7. Show that

$$\frac{\partial(u,v)}{\partial(x,y)} * \frac{\partial(x,y)}{\partial(u,v)} = 1$$

8. Discuss the maxima and minima of the following function

$$f = x^3y^2(1 - x - y)$$

9. Expand the function $e^x \log(1 + y)$ in Taylor series at point (0,0)

10. Show that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3}\, dy \neq \int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3}\, dx$$

11. Prove that it $c > 1$

$$\int_0^\infty \frac{x^c}{c^x}\, dx = \frac{\sqrt{c+1}}{(\log c)c+1}$$

12. Find the volume of the Para boloid generated b revolution about the x-axis of the parabole $y^2 = 4ax$ from $x = 0$ to $x = h$

13. If $V = e^{xyz}(i + j + k)$ find curl v

14. Prove div $(axb) = b.\,cure\ a - a.$ cure b

15. Show that

$$\int_s (axi + byj) + (zk).\hat{n}ds = \frac{4}{3}\pi(a + b + c)$$

Where S is the surface of the sphere $x^2 + y^2 + z^2$

Section B

Note: Attempt any three questions. Each question ca marks.

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Verify stoke's theorem when

$F = (2x - y)i - yz^2 j - y^2 z k$

Where S is the Upper half of sphere $x^2 + y^2 + z^2 = 1$ and ε is boundary

Show that the Larger of two areas into which the circle $x^2 + y^2 = 6ya^2$ is divided by the parabola $y^2 = 12ax$ is $\frac{16}{z} a^2(8\pi - \sqrt{3})$

Apply Pirichlel's integral to find the moment of inertia about the z axis of an octant of the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Find the minimum value of

$$x^2 + y^2 + z^2 \text{ give that } ax + by + cz = p$$

If $y = e^{a \sin^{-1} x}$

Prove that

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$

Find the characteristic equation of the matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Verify that it satisfied by A and hence obtain A^{-1}